A quest for a lower bound for the Hedetniemi Conjecture

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Abstract

The *direct product* $G \times H$ of a graph G and a graph H is defined as that graph whose vertex set is $V(G \times H) = V(G) \times V(H)$, and any two of its vertices (a_1, b_1) and (a_2, b_2) are adjacent in $G \times H$ if and only if a_1 is adjacent to a_2 in G, and b_1 is adjacent to b_2 in H. In 1966 Hedetniemi conjectured that, for all finite graphs G and H, $\chi(G \times H) = min\{\chi(G), \chi(H)\}$. It is well known, and easy to show, that $\chi(G \times H)$ is bounded above by $min\{\chi(G), \chi(H)\}$. It was shown by El-Zaher and Sauer that if G and H are such that $\chi(G), \chi(H) \geq 4$, then $\chi(G \times H) \geq 4$. In pursuit of a more suitable lower bound, Duffus and Sauer managed to show that if a special case of the Hedetniemi Conjecture holds, then the chromatic number of the direct product of any two n-chromatic graphs is at least $n/2$. Under the same assumption we show that this lower bound can be improved to $n - 1$. When the conditions pertaining to this special case are relaxed the lower bound is $n - 2$.

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